Such estimators also called as extreme value volatility estimators are of two types: Method of Moments estimators or the Maximum likelihood estimators (Parkinson (1980),Garman and Klass (1980),Ball and Tourus (1984), Brandt and Jones (2006), Rogers and Satchell (1991),Kunitomo (1992),Magdon and Atiya (2003), Yang and Zhang (2000) and Shu and Zhang (2006)). The ML estimators have a basic drawback that they cannot be expressed in closed form and also suffers from bias due to potential misspecification of the data generating process.Alizadeh, Baandt and Diebold (2002) proved that empirically, numerically and theoretically, such extreme value volatility estimators based on high-low prices are efficient and approximately Gaussian and hence robust to microstructure noise.

One drawback of all the above mentioned extreme value volatility estimators is that they assume that the underlying price process follow a geometric Brownian motion, making it complex to understand and implement.

Andersen, Bollersler and Diebold (2001), Hansen and Lunde (2004), Zhang, Mykland and Ait-Sahalia (2005), Mykland (2005), Martens and Dijk (2007) etc. used high frequency return data to estimate the realized volatility. Even though the high-frequency data help to capture the time-varying volatility more closely, however, for many assets, the high-frequency price data are not available. Moreover, it consumes a lot of time and money to collect and clean the data for use.

With regards to a correlation or covariance estimator, the widely followed approach was based on the standard sample correlation formula using daily returns data. However, Zimmerman , Zumbo and Williams (2003) proved that such an estimator will be biased depending on the underlying distribution. Even then, we can see that there is a dearth in literature on range based covariance or correlation estimators based on daily OHLC prices. Rogers and Shepp (2006) proposed a method to compute the range correlation from linear combination of the daily OHLC pricesfor two correlated Brownian motions for two stocks.Further, Brandt and Diebold (2006) and Brunetti and Lildholdt (2002)looked at estimating the covariance of foreign exchange rates. One of the major drawbacks of both these studies was that they made strong assumptions about the specification of the underlying price process and further limited the applicability of their estimator to specific asset classes such as foreign exchange rates.

Rogers and Zhou (2008) developed a new method of estimating the correlation of a pair of correlated Brownian motions based on the daily high and low stock prices. However, even though it was twice as efficient as the usual volatility estimator, one of the main drawbacks of their estimator was that their correlation estimator had to be within a range of $-1 \leq \rho \leq +1$. Moreover, their assumption of correlated Brownian motions is too restrictive.

Some studies further extended the Rogers and Zhou (2008) estimator. S Sepp (2011) undertook a study to model correlation of stock returns to estimate the volatility of index using a single factor model with mean reversion under the assumption that all correlations are equal. The study showed that adding a mean reversion term improved the predictive capacity of the model. Park (2014) proposed a new method to improve the performance from the data that, the estimator is upward biased for *Cov(b1+c1,y)* and downward biased for *Cov(b2+c2,x)* relative to *Old Cov*.

To summarize the data, we can infer from the above findings that the intraday high and low of futures vary much more widely when compared to spot, suggesting that intraday volatility of futures markets is much more than that of spot market. Futures react faster to information than spot index. This may be due to the fact that futures can be traded as a contract easily, whereas spot index is a basket of stocks, which have to be, traded individually making it expensive and difficult to execute.This may also be an explanation for why future markets lead the spot.

VI. Conclusion

In this paper, we explore an alternative way to estimate the level of covariance between two asset classes having some correlation (ρ). We propose a new covariance estimator *New Cov* based on the daily opening, high, low and closing prices. We prove theoretically that this estimator is unbiased for a random walk process relative to the usual estimator *Old Cov*, thus making the estimator generic. We further validate our theoretical result with the help of simulations for a pair of random walks. We then empirically analyze two correlated asset classes, namely: Nifty Futures and Nifty Spot Index over the period ranging from January 2001 to June 2016. We find clear evidence of an upward bias individually for the two asset classes. When we estimate the level of covariance between the two assets using our estimator, we find evidence that intraday high-low of futures fluctuates more widely than spot index. This phenomenon is well captured by the Constant Elasticity of Variance (CEV) specification, as shown through a simulation exercise. This happens because the coefficient δ_x that induces the level dependence effect in volatility in Nifty Futures (Asset X) is greater than δ_y that governs Nifty Spot (Asset Y).

The finding that the intraday volatility of futures markets is much more than that of the spot market is intriguing. This may be due to the fact that futures react faster to information than spot index. Futures can be traded as a contract easily, whereas spot index is a basket of stocks and thus have to be, traded individually. This makes the execution of the orders expensive and difficult. This can as well explain the lead-lag relationship between futures and spot index.

In this paper, we provide an alternative tool to model the underlying specification of any asset price. Our empirical findings clearly reject the random walk process for the indices. This will be of immense help when it comes to pricing of derivatives or any financial claims as the pricing depends upon the specific stochastic process that drives the asset returns. The uniqueness of our approach is that even without using high frequency data; one can make use of the readily available OHLC prices to understand the true characteristics of the data. Moreover, the estimator makes the simplest assumption of a time reversible process such as a random walk, thereby making the estimator very generic and universally applicable.We also explore in depth the level dependence phenomenon and suggest a CEV specification that can capture the same. We also come up with a very interesting set of findings that futures are more volatile than spot indices. This will help in risk management by allowing hedgers and traders to take suitable position in derivative contracts as well as portfolio management.

Notes

1 We also undertook simulations with $\rho_x = \rho_y = 0.2$ for $\rho_{xy} = 0.975$, 0.5 and 0.25 across 1,00,000 paths, N = 10, 20, 30, 40, 50 steps in the CEV process over K = 1 month. We obtained similar results as when $\rho_x = \rho_y = 0.4$

2 Defined as per Bloomberg Database

3 Even though Nifty Index started in 1994, Nifty Futures started in 2000 and data is available for the four prices only from 1st Jan 2001. So we have chosen a uniform data period for both the assets namely: 2001-2016.

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